

# BIHAR MATHEMATICAL SOCIETY

Bhagalpur

## Talent Nature Programme(TNP) Mathematics workshop

### SYLLABUS

(B. Sc , M. Sc, UPSC, NET, GATE & IIT JAM)

- 1.Foundation course training for all applicants of TSTM and TNP.
  - 2.Second stage training only for top 50 students from each level/ sublevel of TNP selected on the basis of examination.
- Candidates can download the syllabus on [www.bmsbihar@gmail.com](mailto:www.bmsbihar@gmail.com).  
The syllabus of foundation stage and second stage are mentioned below.



PAPER-I

#### 1.Linear Algebra:

**Foundation Stage**-Vector spaces over  $\mathbb{R}$  and  $\mathbb{C}$ , linear dependence and independence, subspaces, bases, dimension, linear transformations, rank and nullity, matrix of a linear transformation.

Matrices, operations of matrix algebra, Kinds of matrices, Transpose adjoint and inverse of the matrix, Product of determinants, row and column reduction, echelon form, Rank and inverse of matrix, Symmetric and skew-symmetric, Hermitian and skew-Hermitian, orthogonal and unitary matrices, congruence and similarity, Solutions of consistent Systems of Linear equation by Cramer's rule.

**Second stage**-eigenvalues and eigenvectors, characteristic polynomial, Cayley-Hamilton theorem, Matrix representation of linear transformations, Jordan canonical forms, diagonal forms, inner product space, Gram-Schmidt orthonormalization process, self adjoint operator, orthonormal basis, triangular forms, Jordan forms, Quadratic forms, bilinear and quadratic forms.

#### 2.Calculus

**Foundation Stage**- functions, limits, continuity, differentiability, indeterminate forms ,successive differentiation, partial derivatives, Leibnitz theorem, Total derivatives, , mean value theorem, Tangent and Normal, Curvature, Taylor's theorem with remainders, asymptotes; curvature.

Integration of rational and irrational, Function notion of integral as limit of sum, evaluation of definite integrals, reduction formulae, curve tracing, Areas of curves, Length of curves, Volumes and surface areas of solids of revolution.

**Second stage**-functions of two or three variables, maxima and minima, Lagrange's method of multipliers, Beta and Gamma functions, Jacobian, Fundamental theorem of integral calculus, double and triple integrals, Dirichlet's and Liouville's theorem, Change of order of integration, Differentiation under sign of integration and integration under sign of integration, Areas, surface area using double integral and volumes using triple integral.

### 3. Analytic Geometry:

**Foundation Stage**-Family of straight lines and circles, Standard equation of Parabola, Ellipse and Hyperbola, General equation of second degree, Transformation of rectangular axes.

Cartesian and polar coordinates in three dimensions, second degree equations in three variables, reduction to canonical forms, plane, straight lines, shortest distance between two skew lines.

**Second stage**- General equation of conics and its reduction to normal form, Equation of tangent and normal at a point of conics, equation of chord of contact, pair of tangents and director circle, Polar equation of conics and their properties.

Sphere, Cone, Cylinder, Paraboloid, Ellipsoid, Hyperboloid of one and two sheets and their properties.S

### 4. Ordinary Differential Equations:

**Foundation Stage**-Formulation of differential equations, equations of first order and first degree, integrating factor, Bernoulli's equations, orthogonal trajectory; equations of first order but not of first degree, Clairaut's equation, singular solution, Second and higher order linear equations with constant coefficients, complementary function, particular integral and general solution.

**Second stage**-Second order linear equations with variable coefficients, Homogeneous Equation Higher order, Variation of Parameter, Euler-Cauchy equation; Method of Laplace transformations for solving ordinary differential equations, Power series, Legendre and Bessel functions and their orthogonal properties, Frobenius method, determination of complete solution, Application to initial value problems for second order linear equations with constant

coefficients, variation and parameters, Sturm-Liouville boundary value problems, Green function.

## 5. Vector Analysis and Vector Calculus:

**Foundation Stage**-Scalar and vector fields, Dot and Cross product of two vectors, Scalar triple product of vectors, Vector product of three and four vectors, vector identities and vector equations, Applications of vectors in mechanics.

**Second stage**-Differentiation of vector field of a scalar variable, Gradient, Divergence and Curl in cartesian and cylindrical coordinates, higher order derivatives, line integrals, surface integrals, Green, Stokes and Gauss theorems.

## 6. Theory of equations:

**Foundation Stage**- Division algorithm, greatest common divisors, polynomials, division algorithm derivative, integral, rational, real and complex roots of a polynomial relation between roots and coefficients, repeated roots, elementary symmetric function, fundamental theorem of algebra.

**Second stage**- Evaluation of symmetric functions of roots of cubic and biquadratic equations, solutions of cubic equation by Cardon's method, solution of biquadratic equations by Euler's method, Descartes rule of signs.

## 7. Hydrostatics:

**Foundation stage**-Pressure at a point, Equilibrium of fluids under given system of force. centre of pressure, Equilibrium of floating bodies.

## 8. Topology:

**Foundation Stage**-Metric spaces and their basic properties, open sphere, open set, neighborhoods, closed set, accumulation point, closure and interior, convergence of sequence in a metric space and their properties, Cauchy sequence and complete metric space, continuous mappings, Compactness and their basic properties, finite intersection property, Normed linear space.

**Second stage**- Definition and examples of topological space, closed set, closure, Dense subset, Derived set, Bases and sub-spaces, Continuity of functions and homeomorphism, separation axiom  $T_0$ ,  $T_1$ ,  $T_2$  spaces their characteristics and basic properties, connectedness.

## 9.Set Theory

**Foundation Stage-** Set, Subsets, Power Set, Algebra of Sets, De Morgan's Laws, Cartesian Product of sets, relation, equivalence relation, Definition and examples of partial and total order relation, Countable and uncountable sets, Countability of rational, Real And algebraic number system, Countability of unions.

**Second Stage-**Equivalence relation induced by a partition of a set, Fundamental theorem of equivalence relation, Composition and factorization of mapping, set mapping, countability of rational, real and algebraic number system.

## 10.Probability

**Foundation stage-**Event, Probability of an event, sample space, probabilities a finite sample space, Mutually exclusively events and complementary events, independent events, conditional probability, multiplication theorem, theorem of total probability, Bayes theorem and independents of events.

**Second stage-**Random variables and their probability functions. Mathematical expectation and moment of a random variable, Mean absolute deviation, variance, standard variation, Chebyshev's theorems for a probability distribution and frequency distribution of measurements.



### 1.Algebra:

**Foundation Stage-** Binary operation, Notions of group, Abelian group with examples, groups, subgroups, cyclic groups, cosets, Lagrange's theorem, normal subgroups, quotient groups, homomorphism of groups, automorphism, basic isomorphism theorems, Centre, Normalizer, Conjugacy, class equation, Commutator and commutator sub group

Rings, integral domains, subrings and ideals, integral domain, division ring, polynomial ring, field and their examples.

**Second stage**-Permutation groups, Cayley's theorem, Sylow theorems, homomorphisms of rings, Isomorphism, Kernel of a ring homomorphism,

quotient rings, Fundamental theorem of homomorphism rings, imbedding of a ring and integral domain in a field, characteristics of a field, polynomials over commutative ring, unique factorization domain, principal ideal domains, Euclidean domain, polynomial rings, finite fields, field extension, Galois theorem.

## 2. Real Analysis:

**Foundation Stage**-Real number system as an ordered field with least upper bound property, Dedekind's theory of real numbers, Cantor's construction of real numbers, properties of real numbers sequences, limit of a sequence, Cauchy sequence, completeness of real line, Monotonic function, Continuity and uniform continuity of functions, properties of continuous functions on compact sets.

Infinite series and their convergence, Comparison test, Cauchy root test, Raabe's test, Cauchy condensation test, Integral test, Leibnitz's test, Gauss Test, Kummer's test, de Morgan and Bertrand's test, absolute convergence and rearrangement of series, Pringsheim's theorem, Cauchy's multiplication of series and its convergence.

**Second stage**-Riemann sum and Riemann integral, Improper integral, convergence of an improper integral, comparison tests, fundamental theorems of integral calculus. Uniform convergence of sequence and series of functions, Weierstrass M-test, uniform convergence and continuity, Dini's test, Abel's test, Dirichlet's Test, Uniform convergence and integration, Uniform convergence and differentiation, Weierstrass sequence and series of functions and their pointwise convergence, continuity, differentiability and integrability for sequences and series of functions, Inverse and Implicit theorems, maxima and minima, Ascoli-Arzelà theorem, Contraction mapping principle.

## 3. Complex Analysis:

**Foundation Stage**-Algebra of complex numbers, Continuity, Differentiability, Analytic functions, Cauchy-Riemann equations, Contour integral, Cauchy's theorem, Cauchy's integral formula, Liouville's theorem, Morera's theorem, Taylor's series, Laurent's series.

**Second stage**-Calculus of residue, singularities; Isolated singularity, meromorphic function, Argument Principle, Schwartz lemma, open mapping theorem, Cauchy's residue theorem, Rouché's theorem, fundamental theorem of algebra, contour integration, power series representation of an analytic function, Möbius transformation, Conformal mappings.

#### 4. Operation Research:

**Foundation Stage**-Linear programming problems, basic solution, basic feasible solution and optimal solution; graphical method, convex set, simplex method of solutions.

**Second stage**-Revised simplex method, Infeasible and unbounded linear programming problem, Big-M Method, Two phase method, duality, transportation, assignment problems, Game theory, two person-zero sum games with mixed strategies, Sequencing, Replacement model, Kuhn-Tucker condition for constrained optimization, Wolfe's and Beale's methods, Queuing theory, Poisson probability law, Distribution of inter arrival time, Distribution of time between successive arrivals.

#### 5. Partial differential equations:

**Foundation stage**-Family of surfaces in three dimensions and formulation of partial differential equations, solution of quasi-linear partial differential equations of the first order.

**Second stage**- Linear partial differential equations of the second order with constant coefficients, Lagrange's and Charpit's methods for solving first order solving PDEs, Cauchy's problem for first order PDEs, Monge's Method, Method of separation of variables for Laplace, Heat and wave equations.

#### 6. Numerical Analysis:

**Foundation Stage**-Finite differences, Newton's forward and backward interpolation, Lagrange's interpolation, Hermite and spline interpolation, Numerical methods, solution of algebraic and transcendental equations of one variable by bisection, Secant method, iteration method, order of convergence, Regula-Falsi method, Newton-Raphson methods.

**Second stage**-Solution of system of linear equations by Gaussian elimination and Gauss-Jordan (direct), Gauss-Seidel (iterative), Relaxation Method, Numerical integration, Trapezoidal rule, Simpson's rules, Gaussian quadrature formula. Numerical solution of ordinary differential equations, Euler, Modified Euler and Runge Kutta-methods, Picard's method.

#### 7. Fluid Dynamics:

**Foundation stage**- Lagrangian and Eulerian methods, Equation of continuity, Euler's equation of motion for inviscid flow, Stream-lines, path of a particle, Potential flow, irrotational and rotational motions, Sources and sinks, vortex motion.

**Second Stage**-Navier-Stokes equation for a viscous fluid, Bernoulli's theorem, equation of motion by flux method, equation referred to moving axis, impulsive actions.

## 8. Functional Analysis:

**Foundation Stage-** Complex linear space, normed linear space, completion of a normed linear space, quotient space of normed linear space, Banach space and their definition, properties and examples.

**Second stage-** Inner product space and Hilbert space and their properties and examples, Orthonormal bases, projection theorem, Riesz representation theorem, spectral theorems for self adjoint operators, Cauchy Schwartz inequality, parallelogram law and polarization identity, Hahn-Banach theorem on real linear space, Open mapping theorem and closed graph theorems, Principle of uniform boundness.

## 9. Statistics

Mean, Variance and standard deviation of random variables, Binomial, Poisson and Normal distributions, Correlation and linear regressions.

## 10. Mechanics

**Foundation course-** Coplanar force of system, Condition for equilibrium of particles, Equipollent force system, Reduction of a force system to a force and a couple, Work and Energy principle, Equation of the resultant Principle of virtual work in two dimension

Uniformly accelerated motion, Dynamics S.H.M. Simple Pendulum, Elastics String and springs, Hook's Law, vertical and horizontal vibrations of a particle attached to an elastic strings, Components of velocities and acceleration, Cartesian, radial and transverse, tangential and normal.

**Second Stage-** stable equilibrium, Energy test for stability, Catenary Poinsot's central axis pitch, Null lines, Euler's theorem on displacement of a rigid body with one fixed point, displacement of a rigid body, Motion about a fixed point, angular velocity and linear velocity, General motion of a body, Principle of linear momentum, angular momentum and energy for a rigid body, D'Alembert's principle of general equations of motions of rigid body.

Projectile motion in non-resisted medium, Motion of a particle under central force, Differential equation of central orbit in polar and pedal forms Newton's law of gravitation and planetary orbit, Kepler's laws, Moment of inertia, Parallel axes and perpendicular axes theorem,

